

# Smooth braneworld models possibility in modified gravities

G. P. de Brito\*

*Centro Brasileiro de Pesquisas Físicas (CBPF),*

*Rua Dr. Xavier Sigaud 150, Urca, 22290-180, Rio de Janeiro, RJ, Brazil.*

J. M. Hoff da Silva,<sup>†</sup> P. Michel L. T. da Silva,<sup>‡</sup> and A. de Souza Dutra<sup>§</sup>

*Departamento de Física e Química, Universidade Estadual Paulista,*

*Av. Dr. Ariberto Pereira da Cunha, 333, Guaratinguetá, SP, Brazil.*

It is shown that the consideration of the braneworld consistency conditions within the framework of bulk modified gravities allows for the existence of thick branes in the five-dimensional case with compact extra dimension. In studying the specific consistency conditions in the Brans-Dicke gravity we were able to show that the brane generating scalar field potential is relevant for relaxing the gravitational constraints.

PACS numbers: 11.25.-w, 03.50.-z

## I. INTRODUCTION

The idea that we live in a four-dimensional noncompact Universe (the brane) embedded in a higher dimensional spacetime have received considerable amount of attention in the literature (for some comprehensive reviews see refs. [1–4]). In fact, part of this attention to braneworld scenarios was motivated by the possibility of solving certain problems in particle physics, for instance, the so-called mass hierarchy problem. The Randall-Sundrum (RS) framework provides a very beautiful and simple approach to this problem [5]. In that context, the small ratio between the electroweak scale and the Planck scale appears as a consequence of an exponential suppression in the Higgs mass term, due to gravitational effects in a higher dimensional spacetime endowed with warped geometry.

In the RS model there are two 3-branes (our 3+1 dimensional Universe), with opposite tensions, localized at the ends of an  $S^1/\mathbf{Z}_2$  orbifold. In this framework, the branes appear as singular sources

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\*Electronic address: gustavopazzini@gmail.com

†Electronic address: hoff@feg.unesp.br

‡Electronic address: pmichel@fc.unesp.br

§Electronic address: dutra@feg.unesp.br

in the five-dimensional Einstein field equations. Since then, several extensions of the RS model were proposed. A very interesting approach was that adopted by De Wolfe *et.al.* [6], and independently, by Gremm [7], where the full spacetime is a five-dimensional manifold with warped geometry and, the brane-bulk structure is generated by a scalar field coupled to gravity. In this context, the brane is interpreted as being a domain-wall where the standard model fields are localized in.

It is quite difficult, however, to make any judgment regarding the consistency of many models. A special attention must be taken to the question of whether the spacetime geometry, along with background fields and branes, solve the set of Einstein field equations. In a seminal paper, Gibbons, Kallosh and Linde derived a set of consistency conditions [8], the so-called sum rules, which are helpful to solve the consistency problem, without solving the dynamical equations. Particularly, one important conclusion obtained in Ref. [8] is that braneworld scenarios with compact extra dimensions demands the inclusion of negative tension branes, which is a disappointing result, since such objects are gravitationally unstable. Some time later, Leblond, Myers and Winters generalized the brane world sum rules to an arbitrary number of dimensions [9]. These authors also shown that it is possible to evade the necessity of negative tension brane by increasing the number of dimensions of the compact internal space.

An important result derived from the sum rules [8] was the no-go theorem regarding the impossibility of construction of smooth braneworld scenarios with compact internal spaces. The no-go theorem may be stated as follows: smooth generalizations of the RS scenario without singular sources are inconsistent with compact extra dimension. This conclusion was based in the following result

$$\oint \Phi' \cdot \Phi' = 0, \quad (1)$$

where  $\Phi$  is a scalar field, supposedly used as source for the branes, and prime means derivative with respect to the extra dimension. Note that the only possible way to satisfy the above equations is  $\Phi$  taking constant and, as a consequence, there is no brane-bulk structure.

The main aim of this paper is to show that it is possible to escape from the no-go theorem of the last paragraph using modified gravities. In the last year, Ahmed and Grzadkowski attempted to evade this theorem by using a non-minimal scalar-gravity coupling, nevertheless the result was negative [10]. In fact, a crucial difference between the approach used in Ref. [10] and the presented here is that in the former case the non-minimal coupled scalar field is, at the same time, the scalar field responsible to generate the brane. Therefore, when the gravitational system of equations are to be satisfied along with the  $Z_2$  symmetry, there are too much constraints over the very same field,

and it turns out that only the trivial shape is allowed. In contrast, in our case even when the gravity sector is also implemented by a scalar field, there is another scalar field shaping the brane.

A clue on the use of modified gravities can be read from the net result of Ref. [11]. In Ref. [11] it was shown that the presence of a Gauss-Bonnet term in the five-dimensional lagrangian can be used to, in some cases, turn the braneworld sum rules less severe. On the other hand the functional form of the resulting consistency conditions in [11] is, perhaps, too much complicated, encouraging the searching for different solutions.

In the present work, we revisited the sum rules in the context of  $f(R)$  and Brans-Dicke (BD) gravities showing that in both cases the no-go theorem of the previous paragraph may be relaxed, allowing for the construction of smooth versions of braneworld scenarios in accordance with compact internal spaces. In a sense, given the equivalence of  $f(R)$  and Brans-Dicke theory in some specific cases, the possibility of circumventing the no-go theorem in both theories is expected. On the other hand, as stated, since the equivalence is only valid for specific cases, a full treatment of both cases is necessary. It is interesting to note that the consistency conditions were already incorporated in the context of  $f(R)$  [12] and BD gravities [13, 14], in these works it was shown that both  $f(R)$  and BD theories are suitable to accommodate consistent scenarios without negative tension branes.

This work is organized as follows: in the next Section we attempt to investigate the sum rules when applied to the  $f(R)$  and Brans-Dicke cases. As we shall see, both cases allow for existence of smooth 3-branes in five dimensions. In the last Section we conclude.

## II. SUM RULES FOR BRANEWORLD SCENARIOS

By considering the spacetime as a  $D$ -dimensional manifold endowed with a nonfactorizable geometry, we write the line element as

$$ds^2 = G_{AB}(X)dX^A dX^B = W^2(r)g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(r)dr^m dr^n, \quad (2)$$

where  $W^2(r)$  is the warp factor,  $X^A$  denotes the coordinates of the full  $D$ -dimensional spacetime,  $x^\mu$  stands for the  $(p+1)$  coordinates of the noncompact spacetime (brane), and  $r^m$  labels the  $(D-p-1)$  directions in the internal compact space. The relevant classical action for our purposes takes into account the spacetime dynamics coupled to a scalar field, namely

$$S = S_{gravity} + \int d^D X \sqrt{-G} \left( -\frac{1}{2} \partial_A \Phi \partial^A \Phi - V(\Phi) \right), \quad (3)$$

where we assume that the scalar field has only dependence on the internal space coordinates  $\Phi = \Phi(r^m)$ . From the above action we obtain the following expressions for the energy-momentum

tensor associated with the scalar field source

$$T_{\mu\nu} = -W^2 g_{\mu\nu} \left( \frac{1}{2} \nabla\Phi \cdot \nabla\Phi + V(\Phi) \right), \quad (4)$$

and

$$T_{mn} = \nabla_m \Phi \nabla_n \Phi - g_{mn} \left( \frac{1}{2} \nabla\Phi \cdot \nabla\Phi + V(\Phi) \right). \quad (5)$$

The key point concerning this construction is to relate  $D$ -dimensional geometric objects with the corresponding geometrical objects on the brane and on the internal compact space. It can be seen [8, 9] that if  $\bar{R}$  is the scalar of curvature derived from  $g_{\mu\nu}$  and  $\tilde{R}$  its counterpart in the internal space, then the following relation holds

$$\nabla \cdot (W^\alpha \nabla W) = \frac{W^{\alpha+1}}{p(p+1)} \left[ \alpha (W^{-2} \bar{R} - R_\mu^\mu) + (p - \alpha) (\tilde{R} - R_m^m) \right], \quad (6)$$

where  $R_\mu^\mu = W^{-2} g^{\mu\nu} R_{\mu\nu}$  and  $R_m^m = g^{mn} R_{mn}$  are the partial traces such that  $R = R_\mu^\mu + R_m^m$  and  $\alpha$  is an arbitrary parameter. It must be emphasized that so far we have not specified anything regarding the gravitational theory that we are considering. This physical piece is implemented as far as we choose a dynamical gravitational equation relating the partial traces to the energy-momentum tensor. Another relevant point is that the left hand side of Eq. (6) vanish upon integration over the internal compact space.

### A. Sum rules in $f(R)$ theories

In the case where the bulk is gravity described by  $f(R)$  theories, its dynamical equation is given by [15]

$$F(R) R_{AB} - \frac{1}{2} G_{AB} f(R) + G_{AB} \square F(R) - \nabla_A \nabla_B F(R) = 8\pi G_D T_{AB}, \quad (7)$$

where  $F(R) = df(R)/dR$ . Eq. (7) can also be recasted in the equivalent form below

$$R_{AB} = \frac{1}{F(R)} \left[ 8\pi G_D \left( T_{AB} - \frac{G_{AB}}{D-2} T \right) + \nabla_A \nabla_B F(R) + \frac{G_{AB}}{D-2} \left( \square F(R) - f(R) + R F(R) \right) \right], \quad (8)$$

from which the partial traces are readily computed. Part of the algebraic manipulation hence forward was performed before [12], therefore we shall only enumerate the main steps.

Computing the partial traces from Eq. (8), inserting the result in (6), taking into account  $T_\mu^\mu$  and  $T_m^m$  coming from Eq. (5) and, finally, considering the most appealing case,  $D = 5$  and  $p = 3$ ,

we have

$$\begin{aligned} \alpha \bar{R} \oint W^{\alpha-1} = 8\pi G_5 \oint \frac{W^{\alpha+1}}{F(R)} \left[ (3-\alpha)\Phi' \cdot \Phi' + 2(\alpha+1)V(\Phi) \right] + 4 \oint \frac{W^{\alpha+1} \nabla^2 F(R)}{F(R)} + \\ + (\alpha+1) \oint W^{\alpha+1} \left( R - \frac{f(R)}{F(R)} \right) + (2\alpha+1) \oint \frac{W^{\alpha+1} \nabla^\mu \nabla_\mu F(R)}{F(R)}. \end{aligned} \quad (9)$$

As expected, the General Relativity  $\alpha$ -family of consistency conditions can be recovered from the above expression. The most sharp consistency condition regarding the possibility of smooth branes is given when  $\alpha = -1$ , since it eliminates the contribution coming from the scalar field potential. Hence, setting  $\alpha = -1$  and  $\bar{R} = 0$  (in order to reproduce the observational evidence of a flat four-dimensional universe) we have

$$32\pi G_5 \oint \frac{\Phi' \cdot \Phi'}{F(R)} + 4 \oint \frac{\nabla^2 F(R)}{F(R)} - \oint \frac{\nabla^\mu \nabla_\mu F(R)}{F(R)} = 0. \quad (10)$$

Taking into account that in the present case the bulk scalar of curvature reads

$$R = -\frac{4}{W} \nabla^2 W - \frac{\nabla^2 W^4}{W^4}, \quad (11)$$

we have, as a consequence,  $f(R) \rightarrow f(W)$ ,  $F(R) \rightarrow F(W)$  and  $\nabla_\mu F(R) = \nabla_\mu F(W) = 0$ . Thus, Eq. (10) reduces to

$$\oint \frac{\Phi' \cdot \Phi'}{F(W)} + \frac{1}{8\pi G_5} \oint \frac{\nabla^2 F(W)}{F(W)} = 0. \quad (12)$$

The above result is very interesting, since it provides a route to escape from the no-go theorem mentioned earlier. The inconsistency appearing in Eq. (1), straightforwardly recovered from the equation above, relies on the fact that the left hand side should be positive<sup>1</sup>. However, as we can see, in the context of  $f(R)$  theories, there is another resulting term in the relevant sum rule, allowing for the possibility of smooth braneworld scenarios along with a compact internal space.

## B. The Brans-Dicke case

As previously stated, the equivalence between  $f(R)$  and scalar-tensorial theories is ensured only for very particular choices of the Brans-Dicke parameter. Therefore, in order to investigate whether the no-go theorem is valid in the Brans-Dicke case, we have to adequate our previous approach to this case [13, 14].

The Brans-Dicke dynamics is described by the following equations [16, 17]

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<sup>1</sup> At least for solutions with extra dimensional dependence, which are the ones of our interest.

$$\begin{aligned}
R_{AB} - \frac{1}{2}G_{AB}R &= \frac{8\pi}{\phi}T_{AB} + \frac{\omega}{\phi^2} \left( \nabla_A \phi \nabla_B \phi - \frac{1}{2} \nabla_C \phi \nabla^C \phi G_{AB} \right) \\
&\quad + \frac{1}{\phi} \left( \nabla_A \nabla_B \phi - \frac{8\pi}{(D-1) + (D-2)\omega} T G_{AB} \right)
\end{aligned} \tag{13}$$

and

$$\square \phi = -\frac{\phi}{2\omega}R + \frac{1}{2\phi} \nabla^A \phi \nabla_A \phi, \tag{14}$$

where  $T_{AB}$  is the matter stress-tensor and  $\omega$  is the so-called Brans-Dicke parameter. We reinforce that  $\phi$  is the Brans-Dicke scalar field, not to be confused with the kink shape scalar field which generates the brane. Taking Eqs. (13) and (14) together it is possible to compute the partial traces for this case. In this vein, Eq. (6) for the Brans-Dicke theory reads

$$\begin{aligned}
\nabla \cdot (W^\alpha \nabla W) &= \frac{W^{\alpha+1}}{p(p+1)} \left\{ \alpha W^{-2} \tilde{R} + (p-\alpha) \tilde{R} + \frac{8\pi}{\phi} \frac{1}{[(D-1) + (D-2)\omega]} \right. \\
&\quad \times \left( T_\mu^\mu \left[ (D-p-2)(p-2\alpha) - \alpha(D-p-3)\omega + (p-\alpha)(D-p-1)\omega \right] \right. \\
&\quad \left. \left. + T_m^m (\omega-1) \left[ \alpha(p+1) - (p-\alpha)(p-1) \right] \right) - \frac{\omega}{\phi^2} (p-\alpha) \phi' \cdot \phi' \right\}, \tag{15}
\end{aligned}$$

with the particular assumption that only the bulk respect the Brans-Dicke gravity, disregarding, then,  $\nabla_\mu \phi$  terms. This assumption is quite conceivable, since it does not jeopardize the standard four dimensional gravity. Now, inserting the energy-momentum partial traces and integrating over the internal space, we have

$$\begin{aligned}
\oint W^{\alpha+1} \left\{ \alpha W^{-2} \tilde{R} + (p-\alpha) \tilde{R} + \frac{8\pi}{\phi} \frac{1}{[(D-1) + (D-2)\omega]} \left( -\frac{1}{2} \Phi' \cdot \Phi' [A(p+1) + B(D-p-3)] \right. \right. \\
\left. \left. - V(\Phi) [A(p+1) + B(D-p-1)] \right) - \frac{\omega}{\phi^2} (p-\alpha) \phi' \cdot \phi' \right\} = 0, \tag{16}
\end{aligned}$$

where

$$\begin{aligned}
A &\equiv (D-p-2)(p-2\alpha) - \alpha(D-p-3)\omega + (p-\alpha)(D-p-1)\omega, \\
B &\equiv (p+1)(\omega+1)(2\alpha-p),
\end{aligned} \tag{17}$$

are constant parameters.

We are now in position of particularize the above formalism to the five-dimensional bulk case.

Then, setting  $D = 5$  and  $p = 3$  we find

$$\oint W^{\alpha+1} \left\{ -\frac{8\pi}{\phi} \frac{1}{4+3\omega} \left( \frac{1}{2} \Phi' \cdot \Phi' [12(1+\omega) - 8\alpha - 6\omega(\alpha-1)] + V(\Phi) [12(-1+\omega) + 8\alpha + 6\omega(\alpha-1)] \right) - \frac{\omega}{\phi^2} (3-\alpha) \phi' \cdot \phi' \right\} = 0. \quad (18)$$

There are at least two choices for the parameter  $\alpha$  bringing relevant information about the system.

To begin with, notice that for  $\alpha = -1$  we have

$$\frac{(5+6\omega)}{4+3\omega} \oint \frac{1}{\phi} \Phi' \cdot \Phi' = \frac{10}{4+3\omega} \oint \frac{1}{\phi} V(\Phi) + \frac{\omega}{4\pi} \oint \frac{1}{\phi^2} \phi' \cdot \phi' = 0, \quad (19)$$

which in the limit  $\omega \rightarrow \infty$  ( $\phi \rightarrow cte$ ) leads, as expected, to the same consistency condition given by Eq. (1), expliciting the no-go theorem. Returning to Eq. (19), it is remarkable that we have a relaxing of the consistency condition, in the same spirit of Eq. (12). In the present case, however, we see an odd characteristic coming exclusively from the Brans-Dicke theory, namely, the presence of the scalar field potential in the consistency condition. Whenever the choice  $\alpha = -1$  is made, it eliminates the potential of this consistency condition in the usual (General Relativity) and  $f(R)$  framework. The result encoded in Eq. (19) shows that within the Brans-Dicke theory the potential used to set the scalar field shape is, indeed, relevant for the obtainment of a well defined model. It is also interesting the choice  $\alpha = 3$ , since the  $\phi' \cdot \phi'$  contribution vanishes in this case, leading to

$$\oint \frac{W^4}{\phi} \Phi' \cdot \Phi' = (4\omega + 2) \oint V, \quad (20)$$

expliciting, once again, the relevance of the Brans-Dicke theory for the mitigation of the constraints coming from the braneworld sum rules.

### III. CONCLUDING REMARKS

The necessity of a braneworld scenario composed by smooth branes is widely accepted from the physical point of view. Indeed, the very fact that standard gravitational theories should be replaced above the Planck lenght scale points to the need of branes with some thickness. On the other hand, the outcome obtained from the formulation of the braneworld sum rules is exhaustive: within General Relativity, smooth branes are not allowed in five dimensions (assuming a compact extra dimension).

Revisiting the sum rules, this time with the auspices of two modified gravitational theories,  $f(R)$  and Brans-Dicke, we see that in both cases it is possible to relax the consistency conditions and smooth branes are possible, in principle.

Our framework is mainly based upon that fact that the integral of a total divergence over a compact internal space must be equal to zero by means of the orbifold symmetry. By compact, then, we mean a finite internal space without boundaries. Particularly, this approach sounds reasonable in five dimensions since in this case the internal space is nothing but the extra dimension.

In the particular Brans-Dicke framework, it is also interesting to note that the scalar field potential can contribute to the sum rule allowing for the smooth brane. This is exclusive from the formulation within the Brans-Dicke theory and naturally opens up new possibilities of smooth brane modeling, enriching scenarios constructed in such framework. This possibility, raised in the realm of the Brans-Dicke theory, may be faced as a bonus in braneworld modeling. As we have mentioned, in General Relativity, smooth branes are not allowed in five dimension regardless the scalar field potential. Regardless the model, for short. In this vein, there are plenty of models, respecting the constraint (20) for instance, which can be implemented within Brans-Dicke gravity. We believe that this possibility may encourage model builders outside the General Relativity framework.

### Acknowledgments

ASD and JMHS thanks to CNPq for financial support. GPB thanks to FAPESP for financial support, and PMLTS acknowledge CAPES for financial support.

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